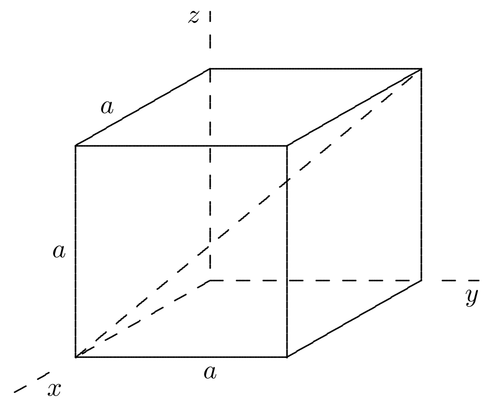
27. (a) There are 4 such lines, one from each of the corners on the lower face to the diametrically opposite corner on the upper face. One is shown on the diagram. Using an xyz coordinate system as shown (with the origin at the back lower left corner) The position vector of the "starting point" of the diagonal shown is $a\,\hat{\mathrm{i}}$ and the position vector of the ending point is $a\,\hat{\mathrm{j}} + a\,\hat{\mathrm{k}}$, so the vector along the line is the difference $a\,\hat{\mathrm{j}} + a\,\hat{\mathrm{k}} - a\,\hat{\mathrm{i}}$.

The point diametrically opposite the origin has position vector $a\,\hat{\mathrm{i}}+a\,\hat{\mathrm{j}}+a\,\hat{\mathrm{k}}$ and this is the vector along the diagonal. Another corner of the bottom face is at $a\,\hat{\mathrm{i}}+a\,\hat{\mathrm{j}}$ and the diametrically opposite corner is at $a\,\hat{\mathrm{k}}$, so another cube diagonal is $a\,\hat{\mathrm{k}} - a\,\hat{\mathrm{i}} - a\,\hat{\mathrm{j}}$. The fourth diagonal runs from $a\,\hat{\mathrm{j}}$ to $a\,\hat{\mathrm{i}} + a\,\hat{\mathrm{k}}$, so the vector along the diagonal is $a\,\hat{\mathrm{i}} + a\,\hat{\mathrm{k}} - a\,\hat{\mathrm{j}}$.



(b) Consider the vector from the back lower left corner to the front upper right corner. It is $a\,\hat{\mathrm{i}} + a\,\hat{\mathrm{j}} + a\,\hat{\mathrm{k}}$. We may think of it as the sum of the vector $a\,\hat{\mathrm{i}}$ parallel to the x axis and the vector $a\,\hat{\mathrm{j}} + a\,\hat{\mathrm{k}}$ perpendicular to the x axis. The tangent of the angle between the vector and the x axis is the perpendicular component divided by the parallel component. Since the magnitude of the perpendicular component is $\sqrt{a^2+a^2} = a\sqrt{2}$ and the magnitude of the parallel component is a, $\tan\theta = \left(a\sqrt{2}\right)/a = \sqrt{2}$. Thus $\theta = 54.7^\circ$. The angle between the vector and each of the other two adjacent sides (the y and z axes) is the same as is the angle between any of the other diagonal vectors and any of the cube sides adjacent to them.

(c) The length of any of the diagonals is given by $\sqrt{a^2+a^2+a^2} = a\sqrt{3}$.